

# Influence of nonlocal damping on the field-driven domain wall motion

H. Y. Yuan,<sup>1</sup> Zhe Yuan,<sup>1,\*</sup> Ke Xia,<sup>1</sup> and X. R. Wang<sup>2,3</sup>

<sup>1</sup>*The Center for Advanced Quantum Studies and Department of Physics,  
Beijing Normal University, Beijing 100875, China*

<sup>2</sup>*Department of Physics, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong*

<sup>3</sup>*HKUST Shenzhen Research Institute, Shenzhen 518057, China*

(Dated: May 11, 2016)

We derive the complete expression of nonlocal damping in noncollinear magnetization due to the nonuniform spin current pumped by precessional magnetization and incorporate it into a generalized Thiele equation to study its effects on the dynamics of the transverse and vortex domain walls (DWs) in ferromagnetic nanowires. We demonstrate that the transverse component of nonlocal damping slows down the field-driven DW propagation and increases the Walker breakdown field whereas it is neglected in many previous works in literature. The experimentally measured DW mobility variation with the damping tuned by doping with heavy rare-earth elements that had discrepancy from micromagnetic simulation are now well understood with the nonlocal damping. Our results suggest that the nonlocal damping should be properly included as a prerequisite for quantitative studies of current-induced torques in noncollinear magnetization.

## I. INTRODUCTION

Gilbert damping,<sup>1</sup> which is spatially local and was introduced to describe the energy dissipation in magnetization dynamics, is a phenomenological parameter of magnetic materials although a microscopic theory based on the spin-orbit interaction and disorder scattering is available.<sup>2</sup> The energy dissipation plays an important role in both current-driven<sup>3</sup> and field-driven<sup>4</sup> magnetization dynamics. For example, in the field-driven domain wall (DW) propagation in a magnetic wire, the propagation speed is proportional to the energy dissipation rate.<sup>4</sup> Thus a comprehensive understanding of dissipation (damping) in magnetic materials is not only fundamentally interesting, but also technologically important since the performance of many spintronic devices such as race-track memory<sup>5</sup> is directly related to the DW propagation speed. In the past several decades, the progress in our understanding of the damping has greatly advanced both theoretically and experimentally. Theoretically, the Gilbert damping of real materials can be calculated by using the torque-correlation model,<sup>6–8</sup> Kubo formalism<sup>9–11</sup> and the scattering approach<sup>12–14</sup> in combination with the first-principles electronic structures. In experiments, the value of Gilbert damping can be measured by ferromagnetic resonance (FMR).<sup>15–19</sup> For a nonuniform magnetic structure (noncollinear magnetization) such as a DW in a ferromagnetic wire, FMR is not applicable and the Gilbert damping is usually extracted via measuring the field-driven DW velocity.<sup>4,20–22</sup> This technique is based on the following general features of the field-driven DW propagation: below a critical field, a DW propagates like a rigid body and its velocity is proportional to the external field and inversely proportional to the Gilbert damping.<sup>23</sup> Surprisingly, the extracted Gilbert damping coefficient of permalloy (Ni<sub>80</sub>Fe<sub>20</sub>) from field-driven DW motion is three times larger than the value of the same material from FMR measurement.<sup>19</sup> The enhanced damping in magnetic DW has been at-

tributed to the surface roughness of a ferromagnetic nanowire in combination with texture-enhanced Gilbert damping arising from spin pumping.<sup>19</sup>

Spin pumping was first proposed to understand the Gilbert damping enhancement in a thin ferromagnetic film in contact with a nonmagnetic metal.<sup>24</sup> A precessional magnetization  $\mathbf{m}(t)$  pumps an electron spin current of polarization  $\mathbf{j}_{\text{pump}}^s \sim \mathbf{m} \times \partial \mathbf{m} / \partial t$  into the nonmagnetic metal that dissipates via spin-flip scattering.<sup>25,26</sup> Spin pumping does not have any observable effect in a homogeneous magnetic structure because the net inflow/outflow spin current anywhere in the system is zero due to a precise cancellation of the pumped spin currents in opposite directions. In noncollinear magnetization, the partial cancellation of the spatially dependent spin pumping gives rise to a nonzero net inflow/outflow  $-\nabla[\mathbf{m}(\mathbf{r}) \times \partial \mathbf{m}(\mathbf{r}) / \partial t]$ . This results in an extra torque that has nonlocal damping in nature, different from the local Gilbert damping due to spin-orbit interaction. The total damping of a nonuniform magnetic structure such as a spin spiral and a DW is enhanced through spin pumping<sup>27–31</sup> or spin wave emission.<sup>32</sup> The enhanced damping in nonuniform magnetic structure of permalloy has been quantitatively calculated from the first-principles, which depends not only on the magnetization gradient, but also on the particular dynamical modes.<sup>33</sup>

The torque due to the pumped spin current from precessional noncollinear magnetization can be decomposed into a longitudinal and a transverse components, which were independently predicted by several groups around the same time. Foros *et al.*<sup>29</sup> and Zhang *et al.*<sup>31</sup> showed that the longitudinal spin current increases the effective damping in spin spirals and DWs. Hankiewicz *et al.*<sup>27,28</sup> discovered that the transverse spin current influences the dissipation of spin waves. Many previous works in literature only include the longitudinal component.<sup>19,34–36</sup> For the field-driven transverse DW motion, the first-principles calculation showed that the transverse and longitudinal spin currents are responsible for the rigid-

body motion below a critical field and the oscillatory motion above the critical field, respectively.<sup>33</sup> For more complicate magnetic structures with noncollinear magnetization, e.g. vortex DWs, how this extra nonlocal damping influences magnetization dynamics is not entirely clear. For example, it was predicted that the longitudinal component of the pumped spin current has no effect on steady-state DW motion.<sup>31</sup> On the other hand, micromagnetic simulation showed that the longitudinal component can slow down the field-driven DW propagation as what was observed in experiments.<sup>19</sup> The inconsistency in the theory and experiment requires a better understanding of the effects of the nonlocal damping on the DW motion.

In this paper, the complete form of the nonlocal damping is derived from physically transparent spin pumping formalism and is incorporated into a generalized Thiele equation (GTE), which describes the steady-state motion of a nonuniform magnetic structure. We focus on the consequences on propagation of different types of DWs resulting from the nonlocal damping. The solutions of the GTE explicitly show the role played by the nonlocal damping in DW motion, where the longitudinal and transverse components are equally important in general. For the steady motion of a DW driven by an external field, the longitudinal component vanishes while the transverse one slows down the DW velocity and increases the Walker breakdown field. The experimentally observed dependence of the DW mobility on the damping parameter, which were systematically lower than the values from micromagnetic simulations, can be understood by including the nonlocal damping. As a prerequisite, quantitative extraction of current-induced torques in noncollinear magnetization dynamics relies on an accurate description of nonlocal damping in analytical models and micromagnetic simulations. The paper is organized as follows. In Sec. II, we derive the nonlocal damping and incorporate it into the GTE. The GTE for both transverse and vortex DWs in a magnetic nanowire is then solved in Sec. III under a static external magnetic field along the wire. The solution is also used to determine the strength of the nonlocal damping from available experimental data. The conclusions are summarized in Sec. IV. In Appendix A, we provide a detailed derivation of the spin pumping and the resulting damping torque of noncollinear magnetization in a ferromagnetic nanowire.

## II. GENERAL FORMALISM

### A. Nonlocal damping torque

We consider a nonuniform magnetic structure in a nanowire as schematically shown in Fig. 1. The  $x$ -,  $y$ -, and  $z$ -axes are along the length, width and thickness directions, respectively. A spin current polarized along  $\sim \mathbf{m}(\mathbf{r}, t) \times \partial_t \mathbf{m}(\mathbf{r}, t)$  is pumped out towards all directions<sup>25</sup> as magnetization  $\mathbf{m}(\mathbf{r}, t)$  changes with time.

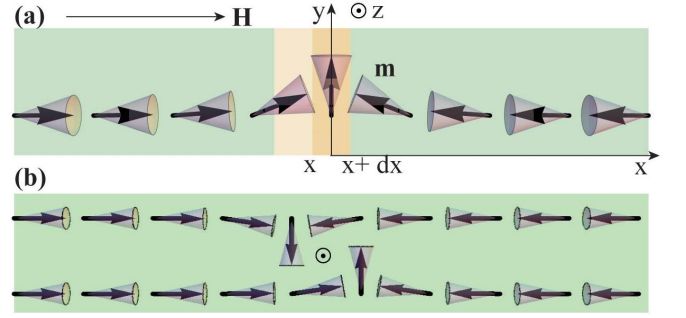


FIG. 1. (a) Top view of a head-to-head transverse DW in a magnetic nanowire. The thick arrows denote the directions of magnetization  $\mathbf{m}$ . The  $x$ -,  $y$ -,  $z$ -axes are along the length, width and thickness directions of the nanowire, respectively. (b) Top view of a vortex DW in a nanowire. The circle in the center denotes the vortex core.

While the detailed derivation is given in Appendix A, the resulting spin current in a noncollinear magnetization reads

$$\begin{aligned} \mathbf{j}_i^s(\mathbf{r}, t) &= -\frac{\hbar}{4\pi} \Gamma_{\uparrow\downarrow} \partial_i [\mathbf{m}(\mathbf{r}, t) \times \partial_t \mathbf{m}(\mathbf{r}, t)] \\ &= -\frac{\hbar}{4\pi} \Gamma_{\uparrow\downarrow} \partial_i \mathbf{m}(\mathbf{r}, t) \times \partial_t \mathbf{m}(\mathbf{r}, t) \\ &\quad - \frac{\hbar}{4\pi} \Gamma_{\uparrow\downarrow} \mathbf{m}(\mathbf{r}, t) \times \partial_i \partial_t \mathbf{m}(\mathbf{r}, t) \\ &\equiv \mathbf{j}_i^{s\parallel}(\mathbf{r}, t) + \mathbf{j}_i^{s\perp}(\mathbf{r}, t). \end{aligned} \quad (1)$$

Here  $i = x, y, z$  denotes the propagation direction of the spin current.  $\Gamma_{\uparrow\downarrow}$  is the (intralayer) spin-mixing conductivity that has the dimension of the inverse of length. Its relation to the conventional spin-mixing conductance is discussed in Appendix A. The longitudinal spin current (LSC)  $\mathbf{j}_i^{s\parallel} = -\frac{\hbar}{4\pi} \Gamma_{\uparrow\downarrow} \partial_i \mathbf{m} \times \partial_t \mathbf{m}$  is always aligned with the local magnetization  $\mathbf{m}$  since both  $\partial_i \mathbf{m}$  and  $\partial_t \mathbf{m}$  are perpendicular to  $\mathbf{m}$ .  $\mathbf{j}_i^{s\perp} = -\frac{\hbar}{4\pi} \Gamma_{\uparrow\downarrow} \mathbf{m} \times \partial_i \partial_t \mathbf{m}$  is the transverse spin current (TSC), which is perpendicular to the local magnetization  $\mathbf{m}$ .

In early works<sup>29,31</sup> with the adiabatic approximation, in which the polarization of the spin current is assumed to align with local magnetization, the transverse spin current is artificially neglected. This approximation is also used in some later works<sup>19,34-36</sup> without a proper justification. Taking permalloy as an example, the first-principles calculations showed that the spin diffusion length and the transverse spin coherent length are 5.5 nm<sup>13</sup> and 13.1 nm,<sup>33</sup> respectively, at low temperature. These lengths are not much smaller than the width of DWs,<sup>37,38</sup> as required for the adiabatic approximation. Thus there are no reasons that the TSC in real materials can be neglected. Indeed, we will see that the TSC substantially influences the magnetization dynamics in the nonuniform magnetic structures, especially for the steady-state DW motion below the critical field.

The dissipative torque generated by the precessional motion of noncollinear magnetization is given by the di-

vergence of the spin current in Eq. (1),

$$\boldsymbol{\tau}_{\text{damping}} = \frac{\gamma}{M_s} \partial_i (\mathbf{j}_i^{\parallel} + \mathbf{j}_i^{\perp}) = \boldsymbol{\tau}_{\text{LSC}} + \boldsymbol{\tau}_{\text{TSC}}, \quad (2)$$

where  $\gamma = g\mu_B/\hbar$  is the gyromagnetic ratio in terms of Landé  $g$  factor and Bohr magneton  $\mu_B$  and  $M_s$  is the saturation magnetization. Einstein summation is assumed in this paper unless it is stated otherwise. The total torque can also be decomposed into the longitudinal and transverse components,  $\boldsymbol{\tau}_{\text{LSC}}$  and  $\boldsymbol{\tau}_{\text{TSC}}$ . Specifically, we have

$$\begin{aligned} \boldsymbol{\tau}_{\text{TSC}} &= -\frac{g\mu_B\Gamma_{\uparrow\downarrow}}{4\pi M_s} \partial_i (\mathbf{m} \times \partial_i \partial_t \mathbf{m}) \\ &= \frac{g\mu_B\Gamma_{\uparrow\downarrow}}{4\pi M_s} [(\mathbf{m} \cdot \partial_i \partial_t \mathbf{m}) \mathbf{m} \times \partial_i \mathbf{m} - \mathbf{m} \times \partial_i^2 \partial_t \mathbf{m}], \end{aligned} \quad (3)$$

and

$$\begin{aligned} \boldsymbol{\tau}_{\text{LSC}} &= -\frac{g\mu_B\Gamma_{\uparrow\downarrow}}{4\pi M_s} \partial_i (\partial_i \mathbf{m} \times \partial_t \mathbf{m}) \\ &= \frac{g\mu_B\Gamma_{\uparrow\downarrow}}{4\pi M_s} \mathbf{m} \times (\mathbf{A} \cdot \partial_t \mathbf{m}), \end{aligned} \quad (4)$$

where  $\mathbf{A}$  is dyadic tensor

$$\mathbf{A} = (\mathbf{m} \times \partial_i \mathbf{m})(\mathbf{m} \times \partial_i \mathbf{m}). \quad (5)$$

Equation (4) reproduces the tensor form obtained in Ref. 31. Alternatively, the second term of  $\boldsymbol{\tau}_{\text{TSC}}$  could also be obtained through an expansion of the Gilbert damping term to the second-order spatial derivative of magnetization<sup>28</sup> or through the phenomenological Landau-Lifshitz-Baryakhtar equation.<sup>39</sup> In a very weak texture like a spin wave, where the precessing magnetization deviates slightly from the equilibrium direction with a cone angle  $\theta$ , the energy dissipation due to the first term of  $\boldsymbol{\tau}_{\text{TSC}}$  is proportional to  $\sin^4 \theta$  while the second term is proportional to  $\sin^2 \theta$ .<sup>33</sup> This is the reason why the first term in Eq. (3) can be neglected in spin wave dynamics.<sup>27,28</sup> It should not be neglected in strong magnetization textures like a transverse DW.<sup>33</sup>

## B. Generalized Thiele equation

To describe the dynamics of noncollinear magnetization, we add the two torques  $\boldsymbol{\tau}_{\text{LSC}}$  and  $\boldsymbol{\tau}_{\text{TSC}}$  into the Landau-Lifshitz-Gilbert (LLG) equation, i.e.

$$\partial_t \mathbf{m} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \partial_t \mathbf{m} + \boldsymbol{\tau}_{\text{LSC}} + \boldsymbol{\tau}_{\text{TSC}}. \quad (6)$$

Here  $\mathbf{H}_{\text{eff}}$  is the effective field that the external applied field, the exchange field, anisotropy and demagnetization fields.  $\alpha$  is the usual local Gilbert damping coefficient. For simplicity, we rewrite the LLG equation in a more compact form as<sup>40</sup>

$$\begin{aligned} \partial_t \mathbf{m} &= -\mathbf{m} \times (\gamma \mathbf{H}_{\text{eff}} - \alpha \partial_t \mathbf{m}) - \eta \nabla^2 (\mathbf{m} \times \partial_t \mathbf{m}), \end{aligned} \quad (7)$$

where we have defined  $\eta \equiv g\mu_B\Gamma_{\uparrow\downarrow}/(4\pi M_s)$  representing the strength of nonlocal damping. Following the Thiele<sup>41</sup> analysis for the field-driven rigid DW motion and by  $\mathbf{m} \times [\text{Eq. (7)}] \cdot \partial_i \mathbf{m}$ , we have

$$\begin{aligned} (\mathbf{m} \times \partial_t \mathbf{m}) \cdot \partial_i \mathbf{m} &= \gamma \mathbf{H}_{\text{eff}} \cdot \partial_i \mathbf{m} - \alpha (\partial_t \mathbf{m}) \cdot (\partial_i \mathbf{m}) \\ &\quad + \eta (\partial_i \mathbf{m} \times \mathbf{m}) \cdot \nabla^2 (\mathbf{m} \times \partial_t \mathbf{m}). \end{aligned} \quad (8)$$

For the rigid DW motion, the magnetization is only the function of  $\mathbf{r} - \mathbf{v}t$ <sup>41,42</sup>, i.e.

$$\theta(\mathbf{r}) = \theta(\mathbf{r} - \mathbf{v}t), \quad \varphi(\mathbf{r}) = \varphi(\mathbf{r} - \mathbf{v}t), \quad (9)$$

where  $\theta$  and  $\varphi$  are, respectively, the polar and azimuthal angles of  $\mathbf{m} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$  in spherical coordinates and  $\mathbf{v}$  is velocity of the magnetic structure. Then the spatial and time derivatives of magnetization can be written as

$$\begin{aligned} \partial_i \mathbf{m} &= \partial_i \theta \hat{\theta} + \sin \theta \partial_i \varphi \hat{\varphi}, \\ \partial_t \mathbf{m} &= \partial_t \theta \hat{\theta} + \sin \theta \partial_t \varphi \hat{\varphi} \\ &= (-\mathbf{v} \cdot \nabla \theta) \hat{\theta} + \sin \theta (-\mathbf{v} \cdot \nabla \varphi) \hat{\varphi}. \end{aligned} \quad (10)$$

Substituting Eq. (10) into Eq. (8), we obtain

$$\begin{aligned} \sin \theta [(\mathbf{v} \cdot \nabla \varphi) \partial_i \theta - (\mathbf{v} \cdot \nabla \theta) \partial_i \varphi] &= \gamma \partial_i (\mathbf{m} \cdot \mathbf{H}_{\text{ext}}) + \alpha \mathbf{v} \cdot (\nabla \theta \partial_i \theta + \sin^2 \theta \nabla \varphi \partial_i \varphi) \\ &\quad - \eta \left[ (-\partial_i \theta \hat{\varphi} + \sin \theta \partial_i \varphi \hat{\theta}) \cdot \nabla^2 (\sin \theta \nabla \varphi \hat{\theta} - \nabla \theta \hat{\varphi}) \right] \cdot \mathbf{v}. \end{aligned} \quad (11)$$

Here  $\mathbf{H}_{\text{ext}}$  is the external magnetic field. Multiplying  $M_s$  to the both sides of Eq. (11) and integrating over the whole nanowire, the generalized Thiele equation becomes

$$\mathbf{F} + \mathbf{G} \times \mathbf{v} + \alpha \mathbf{D} \cdot \mathbf{v} + \eta \mathbf{D}' \cdot \mathbf{v} = 0, \quad (12)$$

where the vectors  $\mathbf{F}$  and  $\mathbf{G}$  and the tensors  $\mathbf{D}$  and  $\mathbf{D}'$

are, respectively,

$$\begin{aligned} \mathbf{F} &= M_s \int \nabla (-\mathbf{m} \cdot \mathbf{H}_{\text{ext}}) d^3 r, \\ \mathbf{G} &= -\frac{M_s}{\gamma} \int (\sin \theta \nabla \theta \times \nabla \varphi) d^3 r, \\ \mathbf{D} &= -\frac{M_s}{\gamma} \int (\nabla \theta \nabla \theta + \sin^2 \theta \nabla \varphi \nabla \varphi) d^3 r, \\ \mathbf{D}' &= \frac{M_s}{\gamma} \int \left( \sin \theta \nabla \varphi \hat{\theta} - \nabla \theta \hat{\varphi} \right) \cdot \nabla^2 \left( \sin \theta \nabla \varphi \hat{\theta} - \nabla \theta \hat{\varphi} \right) d^3 r. \end{aligned} \quad (13)$$

The nonlocal damping appears in the new dissipation term  $\eta \mathbf{D}' \cdot \mathbf{v}$  in Eq. (12). The original Thiele equation<sup>41</sup> is reproduced for  $\eta = 0$ .

### III. FIELD-DRIVEN DW MOTIONS

To explicitly see the effects of nonlocal damping on the DW motion, we apply the GTE (12) to the propagation of transverse and vortex DWs. The analytical results for transverse DWs are compared with micromagnetic simulations of the LLG Eq. (6). We will also use the available experimental data in literature to extract the nonlocal damping coefficient  $\eta$ .

#### A. Transverse DWs

A transverse DW is energetically preferred in relatively narrow and thin nanowires.<sup>37,38</sup> We consider a head-to-head Néel DW in a ferromagnetic nanowire of thickness  $T$  and width  $W$ , as illustrated in Fig. 1(a). The magnetization  $\mathbf{m}(\mathbf{r} - \mathbf{v}t)$  is only a function of  $x - v_x t$ , and  $\mathbf{m}(-\infty) = -\mathbf{m}(+\infty) = \hat{\mathbf{x}}$ . An external field  $\mathbf{H}_{\text{ext}} = H\hat{\mathbf{x}}$  is applied that drives the DW to propagate along  $+x$  direction. Under these conditions, Eq. (13) gives,

$$\begin{aligned} \mathbf{F} &= -TW M_s H \hat{\mathbf{x}} \int dx \partial_x m_x(x) = 2TW M_s H \hat{\mathbf{x}}, \\ \mathbf{G} &= 0, \\ \mathbf{D} &= -\frac{TW M_s}{\gamma} \hat{\mathbf{x}} \hat{\mathbf{x}} \int dx \left[ (\partial_x \theta)^2 + \sin^2 \theta (\partial_x \varphi)^2 \right] \\ &= -\frac{TW M_s}{\gamma} \hat{\mathbf{x}} \hat{\mathbf{x}} \int dx [\partial_x \mathbf{m}(x)]^2, \\ \mathbf{D}' &= \frac{TW M_s}{\gamma} \hat{\mathbf{x}} \hat{\mathbf{x}} \int dx d'_{xx} [\theta(x), \varphi(x)], \end{aligned} \quad (14)$$

where  $d'_{xx}$  is defined as

$$d'_{xx} = \left( \sin \theta \partial_x \varphi \hat{\theta} - \partial_x \theta \hat{\varphi} \right) \cdot \partial_x^2 \left( \sin \theta \partial_x \varphi \hat{\theta} - \partial_x \theta \hat{\varphi} \right). \quad (15)$$

Substituting Eq. (14) into the GTE (12), we find the DW steady-state velocity

$$v_x = \frac{2\gamma H}{\int dx [\alpha (\partial_x \mathbf{m})^2 - \eta d'_{xx}]}. \quad (16)$$

For a Néel DW centered at  $x_0$  and with a width  $\lambda$ , the polar and azimuthal angles of the magnetization are given by the Walker profile,<sup>23</sup>

$$\begin{aligned} \theta(x) &= \frac{\pi}{2}, \\ \varphi(x) &= \pi - \arccos \left[ \tanh \left( \frac{x - x_0}{\lambda} \right) \right]. \end{aligned} \quad (17)$$

The field-driven DW velocity of Eq. (16) can be explicitly calculated,

$$v_x = \frac{\gamma H \lambda}{\alpha + \eta / (3\lambda^2)}. \quad (18)$$

The effective damping of the Walker DW with the nonlocal damping is  $\alpha_{\text{eff}} = \alpha + \eta / (3\lambda^2)$  in agreement with the calculated in-plane damping of transverse DWs using the first-principles scattering theory.<sup>33</sup> The DW propagation is slowed down by the nonlocal damping for a given field.

The so-called Walker breakdown  $H_W$ ,<sup>23</sup> above which the solution for a rigid DW motion does not exist, is  $H_W = \alpha K_z / (\mu_0 M_s)$ <sup>4,23</sup> in a one-dimensional model and in the absence of the nonlocal damping, where  $K_z$  is the total magnetic anisotropy energy along the hard axis including both the magnetocrystalline and shape anisotropy. In the presence of the nonlocal damping, the effective Gilbert damping of a Walker DW becomes  $\alpha + \eta / (3\lambda^2)$  and the Walker breakdown field becomes larger,

$$H_W = \frac{K_z (3\alpha + \eta / \lambda^2)}{3\mu_0 M_s}. \quad (19)$$

The corresponding velocity at the breakdown field can be calculated from Eq. (18) and Eq. (19),

$$v_x^W = \frac{\gamma K_z \lambda}{\mu_0 M_s}, \quad (20)$$

which is not affected by the nonlocal damping.

The LLG equation (6) is numerically solved for a transverse DW in a nanowire of 4 nm thick and 16 nm wide under an external field. The system is discretized into uniform meshes of size 4 nm. The permalloy parameters are used with the saturation magnetization  $M_s = 8 \times 10^5$  A/m, the exchange stiffness  $A = 1.3 \times 10^{-11}$  J/m, the in-plane crystalline anisotropy (along  $x$ )  $K_c = 500$  J/m<sup>3</sup> and Gilbert damping  $\alpha = 0.01$ . The shape anisotropy is implicitly taken into account via including the dipole-dipole interaction in the simulation. Using the above material parameters, we obtain the static DW width  $\lambda \approx 12.6$  nm. When an external field is applied, the width decreases<sup>23</sup> and approaches 9 nm near the breakdown field  $H_W$ .

Figure 2(a) shows the field-dependence of DW propagation velocity  $v_x$  along the  $+x$  direction. At a low external field,  $v_x$  is proportional to  $H$ , following Eq. (18) (straight lines). For the external field larger than 20 Oe, the numerical velocity is slightly lower than the analytical value. This is because the Walker solution becomes unstable as the field is close to the breakdown field and spin wave emission in this regime would further slow down the DW propagation.<sup>32,43</sup>

The critical field  $H_W$  that is obtained from numerical simulations is plotted in Fig. 2(b) as a function of  $\eta$ . The linear relation in Eq. (19) is reproduced and a linear least squares fitting yields the intercept  $H_W(\eta = 0) = 25.9 \pm 0.3$  Oe and the slope  $H_W / \eta = 10.8 \pm 0.3$  Oe/nm<sup>2</sup>. Using Eq. (19), the values of both the intercept and slope consistently lead to the effective anisotropy energy  $K_z = (2.1 \pm 0.1) \times 10^5$  J/m<sup>3</sup>. Then we are able to calculate the analytical value of the DW velocity at the breakdown field using Eq. (20),  $v_x^W = 414$  m/s, as shown by the red

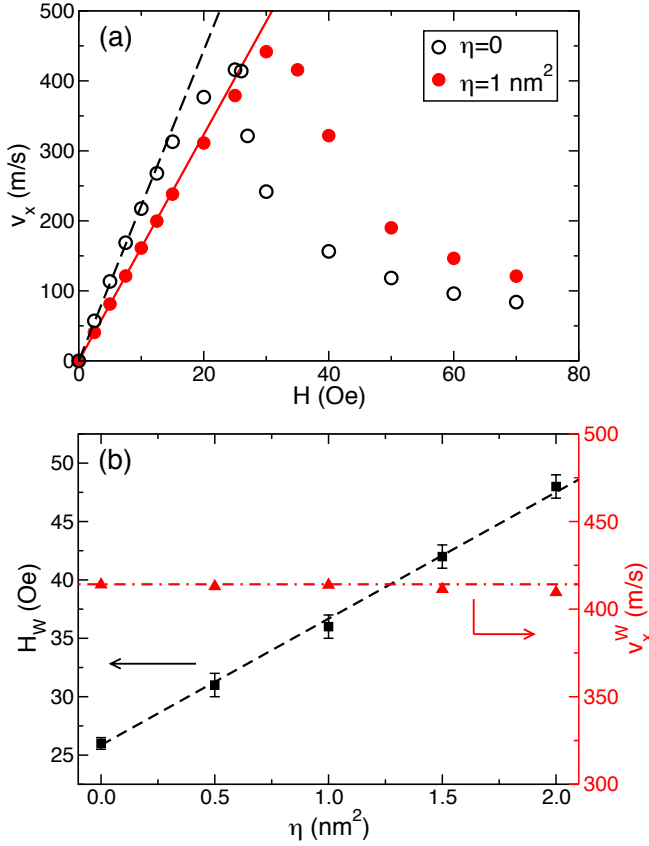


FIG. 2. (a) The velocity of a transverse DW as a function of the external field from numerical simulations without the non-local damping (black open circles for  $\eta = 0$ ) and with  $\eta = 1.0 \text{ nm}^2$  (red solid circles). The black dashed and red solid lines illustrate Eq. (18) with  $\eta = 0$  and  $\eta = 1.0 \text{ nm}^2$ , respectively. (b) The Walker breakdown field  $H_W$  (left axis) and the velocity of DW motion  $v_x^W$  at the Walker breakdown field as a function of the nonlocal damping strength  $\eta$ . The black dashed line is a linear least squares fitting to the black solid squares, which gives the total anisotropy  $K_z = (2.1 \pm 0.1) \times 10^5 \text{ J/m}^3$ . The red dashed-dotted line is the calculated velocity at the breakdown field using Eq. (20) with the fitted  $K_z$ .

horizontal dashed-dotted line in Fig. 2(b). The values of  $v_x^W$  obtained from the numerical simulation plotted in Fig. 2(b) are in very good agreement with the analytical value.

### B. Vortex DWs

A vortex DW is energetically more stable in relatively wide and thick nanowires.<sup>37,38</sup> Here a vortex DW is modeled by an inner vortex core with out-of-plane magnetization and an outer curling structure.<sup>44</sup> The spatial dependence of the polar and azimuthal angles can be ana-

lytically described as<sup>45,46</sup>

$$\theta(x, y) = \begin{cases} 2 \arctan\left(\frac{r}{r_c}\right), & r \leq r_c, \\ \frac{\pi}{2}, & r > r_c, \end{cases}$$

$$\varphi(x, y) = \arg[(x - x_0) + i(y - y_0)] + \frac{\pi}{2}, \quad (21)$$

where  $(x_0, y_0)$  is the center of the vortex core and  $r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ .  $r_c$  is the radius of vortex core that is comparable to the exchange length ( $\sim 5 \text{ nm}$  for permalloy).

The motion of a vortex DW under an external field  $\mathbf{H}_{\text{ext}} = H\hat{\mathbf{x}}$  can be described by applying the GTE (12). Because  $r_c$  is much smaller than the outer curling structure (several tens to hundreds of nanometers) of a vortex DW, the dominant contribution to  $\mathbf{D}$  and  $\mathbf{D}'$  comes from the spins in the curling structure,

$$\begin{aligned} \mathbf{D} &= -\frac{M_s}{\gamma} \int (\sin^2 \theta \nabla \varphi \nabla \varphi) d^3r \\ &= -\frac{TM_s}{\gamma} \int \left( \frac{\partial \mathbf{m}}{\partial x} \right)^2 dx dy (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}) \\ &= -\frac{\pi TM_s}{\gamma} \ln \frac{W}{2r_c} (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}). \end{aligned} \quad (22)$$

Here we have used  $W/2$  as the outer radius of the vortex DW. The tensor  $\mathbf{D}'$  is difficult to derive analytically due to the high order spatial derivatives of  $\theta$  and  $\varphi$  but it is well converged numerically,

$$\mathbf{D}' = -\frac{3.3 TM_s}{\gamma r_c^2} (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}). \quad (23)$$

Similarly, it is a reasonable approximation to neglect the contribution of the vortex core to force  $\mathbf{F}$ . So we have

$$\begin{aligned} \mathbf{F} &= -TM_s H \int \nabla (\sin \theta \cos \varphi) d^2r \\ &\approx TM_s H \int_0^{2\pi} \int_{r_c}^R \left( \frac{\sin^2 \phi}{r}, \frac{-\cos \phi \sin \phi}{r} \right) r dr d\phi \\ &= \frac{\pi W TM_s H}{2} \hat{\mathbf{x}}. \end{aligned} \quad (24)$$

Following He *et al.*,<sup>46</sup> we add a restoring force that is linear in the transverse displacement  $\delta y$  of the vortex core from the nanowire center,  $\mathbf{F}^{\text{re}} = -\hat{\mathbf{y}}\kappa\delta y$ , where the coefficient  $\kappa$  depends on the magnetic anisotropy and the equilibrium width of the vortex DW.<sup>46</sup>

For the gyrovector  $\mathbf{G}$ , the spins outside the vortex core do not contribute because of  $\theta = \pi/2$ . The integral over the vortex core can be analytically evaluated as<sup>46</sup>

$$\mathbf{G} = -\frac{2\pi TM_s}{\gamma} \hat{\mathbf{z}}. \quad (25)$$

Substituting the expressions of  $\mathbf{F}$ ,  $\mathbf{G}$ ,  $\mathbf{D}$  and  $\mathbf{D}'$  into



GTE (12), the coupled equations become

$$\begin{aligned} \frac{\gamma HW}{2} + 2v_y - \left( \alpha \ln \frac{W}{2r_c} + \frac{3.3\eta}{\pi r_c^2} \right) v_x &= 0, \\ -\frac{\gamma \kappa \delta y}{\pi T M_s} - 2v_x - \left( \alpha \ln \frac{W}{2r_c} + \frac{3.3\eta}{\pi r_c^2} \right) v_y &= 0. \end{aligned} \quad (26)$$

Under an external field, the vortex core moves both along the field direction (longitudinal) and transverse to the field ( $y$  direction) due to the gyro force. Then the vortex structure is deformed with the core displacement  $\delta y$  (from the nanowire centre). At steady-state motion, the gyro effect is balanced by the restoring force  $\mathbf{F}^{\text{re}}$ , i.e.  $v_y = 0$ . Then the vortex core only moves in the longitudinal direction and the velocity can be obtained,

$$v_x = \frac{\gamma H \frac{W}{2}}{\alpha \ln \frac{W}{2r_c} + \frac{3.3\eta}{\pi r_c^2}} \equiv \frac{\gamma H \lambda_{\text{eff}}}{\alpha_{\text{eff}}}, \quad (27)$$

where the effective width is defined as

$$\lambda_{\text{eff}} \equiv \frac{W}{2 \ln[W/(2r_c)]}, \quad (28)$$

and the effective damping is

$$\alpha_{\text{eff}} \equiv \alpha + \frac{3.3\eta}{\pi r_c^2 \ln[W/(2r_c)]}. \quad (29)$$

According to Eq. (29), the nonlocal damping enhances the effective damping of a vortex DW. The enhancement depends on the strength of nonlocal damping ( $\eta$ ), the size of vortex core ( $r_c$ ) and the width of nanowire ( $W$ ).

Similar to the case of transverse DWs, the Walker breakdown field increases in the presence of the nonlocal damping. For steady-state motion with  $v_y = 0$ , the transverse displacement of the vortex core is given by

$$\delta y = -\frac{2\pi T M_s H \lambda_{\text{eff}}}{\alpha_{\text{eff}} \kappa}. \quad (30)$$

Equation (30) indicates that the transverse displacement increases with the external magnetic field. When the vortex core reaches the edge of the nanowire  $\delta y = -W/2$ , one has the Walker breakdown field

$$H_W = \frac{\alpha_{\text{eff}} \kappa W}{4\pi T M_s \lambda_{\text{eff}}}. \quad (31)$$

With the nonlocal damping, the breakdown field increases and is proportional to the effective damping  $\alpha_{\text{eff}}$ . The longitudinal velocity at the breakdown field is independent of the nonlocal damping as in the case of transverse DWs.

Determining the value of  $\eta$  from field-driven DW motion and Eq. (27) is not straightforward due to the inhomogeneity of samples. The disorder and surface roughness increase effectively the Gilbert damping by adding a factor  $\alpha_R$ . Both  $\alpha_R$  and the nonlocal damping slow down the field-driven DW propagation.<sup>47,48</sup> Weindler *et al.*

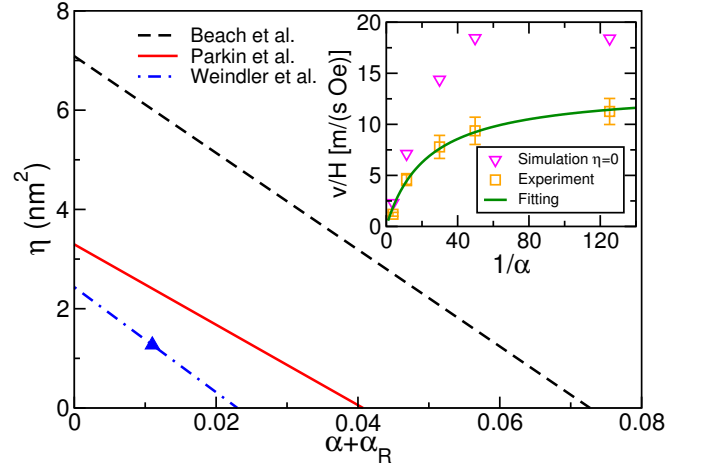


FIG. 3. Calculated  $\eta$  as a function of Gilbert damping parameter including the contributions from spin-orbit interaction  $\alpha$  and roughness  $\alpha_R$ , for the experimental DW velocities driven by an external field taken from Ref. 49 (black dashed line), Ref. 5 (red solid line) and Ref. 19 (blue dashed-dotted line). In Ref. 19,  $\alpha + \alpha_R = 0.011$  is determined by fitting experimental data to the simulations with various DW pinning strengths, and the corresponding nonlocal damping strength  $\eta = 1.3 \text{ nm}^2$  is obtained<sup>50</sup> (blue solid triangle). Inset: Gilbert damping dependence of DW mobility of Ho-doped permalloy nanowires. The orange open squares are experimental data and the magenta open triangles are the simulation without  $\eta$ .<sup>22</sup> The green solid line shows a least squares fitting using Eq. (27), which yields the upper bound of the nonlocal damping strength  $5.1 \pm 1.6 \text{ nm}^2$ .

*al.* measured the Gilbert damping of collinear permalloy  $\alpha = 0.008$  and determined  $\alpha_R = 0.003$  by comparing the experimental and simulated depinning magnetic fields. By using the measured effective damping for the field-driven DW motion  $\alpha_{\text{eff}} = 0.023$ , the nonlocal damping parameter in the system of Ref. 19 is  $\eta = 1.3 \text{ nm}^2$  that is denoted by the blue solid triangle in Fig. 3. We can also estimate  $\eta$  by using the experimental data from Refs. 5 and 49. The result is plotted in Fig. 3 as a function of the Gilbert damping  $\alpha + \alpha_R$ . The inset of Fig. 3 is the experimental data (orange open squares) of the DW mobility for different  $\alpha$  obtained by Moore *et al.*<sup>22</sup> They tuned the value of  $\alpha$  by doping permalloy with a rare-earth element Holmium. Micromagnetic simulation in the absence of the nonlocal damping results in a significantly larger mobility (magenta empty triangles) than the measured values.<sup>22</sup> We use Eq. (27) to fit the experimental data and perfectly reproduce the measured mobility as a function of  $\alpha$ . Since we assume  $\alpha_R = 0$  in the fitting, the only fitted parameter  $\eta = 5.1 \pm 1.6 \text{ nm}^2$  corresponds to the upper bound of the nonlocal damping strength.

The values of  $\eta$  can also be extracted from the wave vector dependence of the spin wave damping measured via FMR.<sup>51–53</sup> The coexisting mechanisms such as the eddy current and the radiative damping result in complexities in determining  $\eta$ . Nembach *et al.*<sup>52</sup> found that

$\eta = 1.4 \text{ nm}^2$  in permalloy nanodisks. Li and Bailey<sup>51</sup> measured permalloy, cobalt and CoFeB alloy and obtained the value  $\eta = 0.11 \pm 0.02 \text{ nm}^2$  for permalloy. Later Schoen *et al.*<sup>53</sup> discussed the contribution of radiative damping and determined that  $\eta$  of permalloy from the experimental measurement is less than  $0.045 \text{ nm}^2$ . All these experiments were performed at room temperature while first-principles calculation at low temperature found  $\eta = 0.016 \text{ nm}^2$  in permalloy but it could be significantly enhanced by two orders of magnitude, up to  $5.9 \text{ nm}^2$ , because of the finite propagating length of transverse spin currents.<sup>33</sup>

In the derivation of the GTE (12), we have included both the transverse and longitudinal torques [Eq. (3) and Eq. (4)] due to the spin pumping. For the steady-state DW motion described by Eq. (10), the longitudinal component  $\tau_{\text{LSC}} = -\eta \partial_i (\partial_i \mathbf{m} \times \partial_t \mathbf{m})$  vanishes because  $\partial_i \mathbf{m}$  is aligned with  $\partial_t \mathbf{m}$ . In our numerical simulation for transverse DWs, it is confirmed that  $\tau_{\text{LSC}}$  alone did not change the DW velocity below breakdown field, consistent with conclusion in Ref. 31 where the expression of  $\tau_{\text{LSC}}$  was derived.

In contrast, Weindler *et al.*<sup>19</sup> considered only  $\tau_{\text{LSC}}$  in their micromagnetic simulations with  $\eta = 0.07 \text{ nm}^2$  and found the texture-enhanced damping in the steady-state DW motion. In micromagnetic simulations, there might be higher order effects that eventually affect the DW mobility. On the other hand,  $\tau_{\text{RSC}}$  can have remarkable influence on the field-driven DW velocity, as predicted by Eq. (27). This is partly the reason why we extracted a different  $\eta = 1.3 \text{ nm}^2$  by using the same experimental data in Sec. III B. It suggests that  $\tau_{\text{RSC}}$  must be properly included to extract a reliable value of  $\eta$ .

#### IV. CONCLUSIONS

We have derived the nonlocal damping torque originated from the pumped spin current by the precessional noncollinear magnetization. This nonlocal damping torque consists of a longitudinal and a transverse components that both depend on the magnetization gradient and inevitably affects the noncollinear magnetization dynamics. We derive a generalized Thiele equation (12) for the field-driven steady-state DW motion under the influence of the nonlocal damping. For both transverse and vortex DWs in ferromagnetic nanowires, the transverse component of the nonlocal damping slows down the DW propagation and increases the Walker breakdown field. The analytical results are further confirmed by numerical simulations for transverse DWs. In addition, our result compares well the experimentally reported damping dependence of vortex DW mobility under an external magnetic field, while the LLG equation without the nonlocal damping significantly overestimates the DW velocity.

The nonadiabatic spin transfer torque  $\beta$  is one of the key parameters in current-driven DW motion because

the velocity is proportional to  $\beta$  and inversely proportional to the damping  $\alpha$ . Previous works in literature extracted  $\beta$  value from experimentally measured DW velocity by assuming a constant  $\alpha$  in a DW. Thus the influence of the nonlocal damping revealed in this paper may explain the large spread in the extracted  $\beta$  values for permalloy.<sup>54–57</sup> The measurement of current-induced torques due to Rashba and Dzyaloshinskii-Moriya interactions in noncollinear magnetization is usually performed by comparing the experimental observation and micromagnetic simulation. As a prerequisite, the nonlocal damping has to be appropriately included in the simulations while the damping form may be more complicated because of the complex interactions.<sup>58</sup>

#### Appendix A: Spin pumping and the damping torque in a noncollinear ferromagnetic nanowire

In this appendix, we derive spin pumping Eq. (1) and the resulting damping torque Eq. (2) exerted on the local magnetization in a noncollinear ferromagnetic nanowire. Without loss of generality, we consider a one-dimensional case with the magnetization  $\mathbf{m}(x, t)$ , in which a small segment of magnetization centered at position  $x$  can be approximated by a local collinear magnetization  $\mathbf{m}(x)$ . As  $\mathbf{m}(x)$  varies in time, it can pump out a spin current

$$\mathbf{j}_{\text{pump}}^s(x) = \frac{\hbar}{4\pi} g_{\uparrow\downarrow} \mathbf{m}(x) \times \partial_t \mathbf{m}(x), \quad (\text{A1})$$

where  $g_{\uparrow\downarrow}$  is the conventional spin-mixing conductance characterizing the magnitude of spin pumping.<sup>25</sup> This pumped spin current flows both forward and backward, as schematically shown by the blue arrows in Fig. 4. Since the magnetization in the nanowire is not uniform, the spin current pumped forward by the segment at position  $x$  is different from that pumped backward by the next segment at position  $x + \Delta x$ . The incomplete cancellation

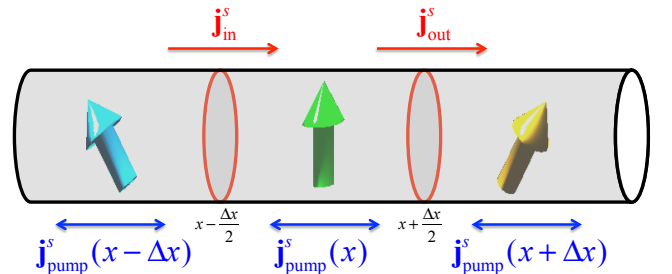


FIG. 4. Sketch of spin pumping in a ferromagnetic nanowire with noncollinear magnetization. A segment of magnetization pumps a spin current  $\mathbf{j}_{\text{pump}}^s(x) = \frac{\hbar}{4\pi} g_{\uparrow\downarrow} \mathbf{m}(x) \times \partial_t \mathbf{m}(x)$ . The net spin current flowing into (out of) the segment at  $x$  is given by the backward (forward) derivative of the pumped spin current. The torque exerted on the segment of magnetization at  $x$  is then defined as the net spin current that is absorbed by the segment of magnetization, as formulated by Eq. (A8).

of these two pumped spin currents gives rise to a net spin current across the cross section at  $x + \Delta x/2$ , which reads

$$\begin{aligned} \mathbf{j}^s \left( x + \frac{\Delta x}{2} \right) &= \mathbf{j}_{\text{pump}}^s(x) - \mathbf{j}_{\text{pump}}^s(x + \Delta x) \\ &= \frac{\hbar}{4\pi} g_{\uparrow\downarrow} [\mathbf{m}(x) \times \partial_t \mathbf{m}(x) \\ &\quad - \mathbf{m}(x + \Delta x) \times \partial_t \mathbf{m}(x + \Delta x)] \\ &= -\frac{\hbar}{4\pi} g_{\uparrow\downarrow} \Delta x \partial_x [\mathbf{m}(x) \times \partial_t \mathbf{m}(x)]_{x + \frac{\Delta x}{2}}. \end{aligned} \quad (\text{A2})$$

If we choose  $+x$  as the positive direction,  $\mathbf{j}^s(x + \Delta x/2)$  corresponds to the outflow of the spin current for the segment of magnetization at  $x$ , which will later be referred to as  $\mathbf{j}_{\text{out}}^s$ .  $\mathbf{j}_{\text{out}}^s$  can be rewritten as a superposition of a longitudinal component  $\mathbf{j}_{\text{out}}^{s\parallel}$  that is aligned with  $\mathbf{m}(x)$  and a transverse one  $\mathbf{j}_{\text{out}}^{s\perp}$  that is perpendicular to  $\mathbf{m}(x)$ ,

$$\mathbf{j}_{\text{out}}^{s\parallel} = -\frac{\hbar}{4\pi} g_{\uparrow\downarrow} \Delta x [\partial_x \mathbf{m}(x) \times \partial_t \mathbf{m}(x)]_{x + \frac{\Delta x}{2}}, \quad (\text{A3})$$

$$\mathbf{j}_{\text{out}}^{s\perp} = -\frac{\hbar}{4\pi} g_{\uparrow\downarrow} \Delta x [\mathbf{m}(x) \times \partial_x \partial_t \mathbf{m}(x)]_{x + \frac{\Delta x}{2}}. \quad (\text{A4})$$

In the same manner, we are able to find the net spin current across the cross section at  $x - \Delta x/2$  corresponding to the inflow,

$$\begin{aligned} \mathbf{j}_{\text{in}}^s &\equiv \mathbf{j}^s \left( x - \frac{\Delta x}{2} \right) = \mathbf{j}_{\text{pump}}^s(x - \Delta x) - \mathbf{j}_{\text{pump}}^s(x) \\ &= -\frac{\hbar}{4\pi} g_{\uparrow\downarrow} \Delta x \partial_x [\mathbf{m}(x) \times \partial_t \mathbf{m}(x)]_{x - \frac{\Delta x}{2}}. \end{aligned} \quad (\text{A5})$$

Its longitudinal and transverse components can be respectively written as

$$\mathbf{j}_{\text{in}}^{s\parallel} = -\frac{\hbar}{4\pi} g_{\uparrow\downarrow} \Delta x [\partial_x \mathbf{m}(x) \times \partial_t \mathbf{m}(x)]_{x - \frac{\Delta x}{2}}, \quad (\text{A6})$$

$$\mathbf{j}_{\text{in}}^{s\perp} = -\frac{\hbar}{4\pi} g_{\uparrow\downarrow} \Delta x [\mathbf{m}(x) \times \partial_x \partial_t \mathbf{m}(x)]_{x - \frac{\Delta x}{2}}. \quad (\text{A7})$$

The difference between inflow and outflow spin currents is absorbed by the local magnetization resulting a damping torque

$$\boldsymbol{\tau}_{\text{damping}} = -\frac{\gamma A (\mathbf{j}_{\text{in}}^s - \mathbf{j}_{\text{out}}^s)}{M_s A \Delta x} = \frac{\gamma}{M_s} \partial_x \mathbf{j}^s(x). \quad (\text{A8})$$

where  $A$  is the cross sectional area of the nanowire and the prefactor  $\gamma/(M_s A \Delta x)$  is to convert the torque in the unit of  $\text{s}^{-1}$ . The torque in Eq. (A8) due to absorption of pumped spin current is only determined by the local magnetization gradient, implying that the length scale of magnetization variation, e.g. a DW width or a spin wave length, is much larger than the propagating length of the pumped spin current in the ferromagnetic nanowire. This condition is in general satisfied in disordered ferromagnetic materials, but it is found in permalloy at low temperature that the spin coherent length is up to 13.1 nm, where the nonlocal absorption of the pumped spin current even changes the scaling of the effective damping with the DW width.<sup>33</sup>

While Eq. (A2) and Eq. (A5) are two particular examples, the net spin current in a noncollinear magnetization resulting from spin pumping can be generally expressed as

$$\begin{aligned} \mathbf{j}^s(x, t) &= -\frac{\hbar}{4\pi} g_{\uparrow\downarrow} \Delta x \partial_x [\mathbf{m}(x, t) \times \partial_t \mathbf{m}(x, t)] \\ &= -\frac{\hbar}{4\pi} \Gamma_{\uparrow\downarrow} \partial_x [\mathbf{m}(x, t) \times \partial_t \mathbf{m}(x, t)]. \end{aligned} \quad (\text{A9})$$

Here we define the spin-mixing conductivity  $\Gamma_{\uparrow\downarrow} \equiv g_{\uparrow\downarrow} \Delta x$  that is a proper parameter characterizing the (intralayer) spin pumping in a noncollinear ferromagnetic material. The conventional spin pumping at a ferromagnet-normal metal interface<sup>25</sup> can be recovered from Eq. (A9) by taking the effective thickness of the interface  $\Delta x$ . Notice that  $g_{\uparrow\downarrow}$  has the dimension of the inverse of area and  $\Gamma_{\uparrow\downarrow}$  has the dimension of the inverse of length.

## ACKNOWLEDGMENTS

H. Y. Y. would like to thank Lei Wang for helpful discussions. This work was financially supported by National Basic Research Program of China (Grant No. 2012CB921304), National Natural Science Foundation of China (Grant No. 61376105). X. R. W. was supported by the National Natural Science Foundation of China (Grant No. 11374249) as well as Hong Kong RGC Grants No. 163011151 and No. 605413.

\* Corresponding author: [zyuan@bnu.edu.cn](mailto:zyuan@bnu.edu.cn)

<sup>1</sup> T. L. Gilbert, IEEE Trans. Magn. **40**, 3443 (2004).

<sup>2</sup> V. Kamberský, Czech. J. Phys. **26**, 1366 (1976).

<sup>3</sup> M. D. Stiles and J. Miltat, *Spin dynamics in confined magnetic structures III*, 225-308 (2006).

<sup>4</sup> X. R. Wang, P. Yan, J. Lu, and C. He, Ann. Phys. (N.Y.) **324**, 1815 (2009); X. R. Wang, P. Yan, and J. Lu, EPL **86**, 67001 (2009).

<sup>5</sup> S. S. P. Parkin, M. Hayashi, L. Thomas, Science **320**, 190 (2008).

<sup>6</sup> K. Gilmore, Y. U. Idzerda, M. D. Stiles, Phys. Rev. Lett. **99**, 027204 (2007).

<sup>7</sup> V. Kamberský, Phys. Rev. B **76**, 134416 (2007).

<sup>8</sup> A. Sakuma, Journal of the Physical Society of Japan, **81**, 084701 (2012).

<sup>9</sup> I. Garate, K. Gilmore, M. D. Stiles, and A. H. MacDonald, Phys. Rev. B **79**, 104416 (2009).

<sup>10</sup> H. Ebert, S. Mankovsky, D. Ködderitzsch, and P. J. Kelly, Phys. Rev. Lett. **107**, 066603 (2011).



- <sup>11</sup> S. Mankovsky, D. Ködderitzsch, G. Woltersdorf, and H. Ebert, *Phys. Rev. B* **87**, 014430 (2013).
- <sup>12</sup> A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, *Phys. Rev. Lett.* **101**, 037207 (2008).
- <sup>13</sup> A. A. Starikov, P. J. Kelly, A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, *Phys. Rev. Lett.* **105**, 236601 (2010).
- <sup>14</sup> Y. Liu, A. A. Starikov, Z. Yuan, and P. J. Kelly *Phys. Rev. B* **84**, 014412 (2011).
- <sup>15</sup> C. Kittel, *Phys. Rev.* **73**, 155 (1948).
- <sup>16</sup> K. Kobayashi, N. Inaba, N. Fujita, Y. Sudo, T. Tanaka, M. Ohtake, M. Futamoto, and F. Kirino, *IEEE Trans. Magn.* **45**, 2541 (2009).
- <sup>17</sup> M. C. Langner, C. L. S. Kantner, Y. H. Chu, L. M. Martin, P. Yu, J. Seidel, R. Ramesh, and J. Orenstein, *Phys. Rev. Lett.* **102**, 177601 (2009).
- <sup>18</sup> X. Liu, W. Zhang, M. J. Carter, and G. Xiao, *J. Appl. Phys.* **110**, 033910 (2011).
- <sup>19</sup> T. Weindler, H. G. Bauer, R. Islinger, B. Boehm, J. -Y. Chauleau, and C. H. Back, *Phys. Rev. Lett.* **113**, 237204 (2014).
- <sup>20</sup> T. Ono, H. Miyajima, K. Shigeto, K. Mibu, N. Hosoi, T. Shinjo, *Science* **284**, 468 (1999).
- <sup>21</sup> D. Atkinson, D. A. Allwood, G. Xiong, M. D. Cooke, C. C. Faulkner, and R. P. Cowburn, *Nat. Mater.* **2**, 85 (2003).
- <sup>22</sup> T. A. Moore, P. Möhrke, L. Heyne, A. Kaldun, M. Kläui, D. Backes, J. Rhensius, L. J. Heyderman, J. -U. Thiele, G. Woltersdorf, A. Fraile Rodríguez, F. Nolting, T. O. Mentes, M. Á. Niño, A. Locatelli, A. Potenza, H. Marchetto, S. Cavill, and S. S. Dhesi, *Phys. Rev. B*, **82**, 094445 (2010).
- <sup>23</sup> N. L. Schryer and L. R. Walker, *J. Appl. Phys.* **45**, 5406 (1974).
- <sup>24</sup> S. Mizukami, Y. Ando, and T. Miyazaki, *Jpn. J. Appl. Phys.* **40**, 580 (2001).
- <sup>25</sup> Y. Tserkovnyak, A. Brataas, G. E. W. Bauer, *Phys. Rev. Lett.* **88**, 117601 (2002); *Phys. Rev. B* **66**, 224403 (2002).
- <sup>26</sup> Y. Liu, Z. Yuan, R. J. H. Wesselink, A. A. Starikov, and P. J. Kelly, *Phys. Rev. Lett.* **113**, 207202 (2014).
- <sup>27</sup> E. M. Hankiewicz, G. Vignale, and Y. Tserkovnyak *Phys. Rev. B* **78**, 020404(R) (2008).
- <sup>28</sup> Y. Tserkovnyak, E. M. Hankiewicz, and G. Vignale, *Phys. Rev. B* **79**, 094415 (2009).
- <sup>29</sup> J. Foros, A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, *Phys. Rev. B* **78**, 140402(R) (2008).
- <sup>30</sup> Y. Tserkovnyak and C. H. Wong, *Phys. Rev. B* **79**, 014402 (2009).
- <sup>31</sup> S. Zhang and Steven S. -L. Zhang, *Phys. Rev. Lett.* **102**, 086601 (2009).
- <sup>32</sup> X. S. Wang, P. Yan, Y. H. Shen, G. E. W. Bauer, and X. R. Wang, *Phys. Rev. Lett.* **109**, 167209 (2012).
- <sup>33</sup> Z. Yuan, K. M. D. Hals, Y. Liu, A. A. Starikov, A. Brataas, and P. J. Kelly, *Phys. Rev. Lett.* **113**, 266603 (2014).
- <sup>34</sup> S. Kim, J. -H. Moon, W. Kim, K. -J. Lee, *Current Appl. Phys.* **11**, 61 (2011).
- <sup>35</sup> J. -H. Moon and K. -J. Lee, *J. Appl. Phys.* **111**, 07D120 (2012).
- <sup>36</sup> K. -W. Kim, J. -H. Moon, K. -J. Lee, and H. -W. Lee, *Phys. Rev. Lett.* **108**, 217202 (2012).
- <sup>37</sup> R. D. McMichael and M. J. Donahue, *IEEE. Trans. Magn.* **33**, 4167 (1997).
- <sup>38</sup> Y. Nakatani, A. Thiaville, and J. Miltat, *J. Magn. Magn. Mater.* **290-291**, 750 (2005).
- <sup>39</sup> W. Wang, M. Dvornik, M. -A. Bisotti, D. Chernyshenko, M. Beg, M. Albert, A. Vansteenkiste, B. V. Waeyenberge, A. N. Kuchko, V. V. Kruglyak, and H. Fangohr, *Phys. Rev. B* **92**, 054430 (2015).
- <sup>40</sup> Note that the magnetization magnitude is always fixed in dynamics described by the LLG equation. The spatial and temporal derivatives only act on the transverse components of magnetization, as explicitly guaranteed by Eq. (10).
- <sup>41</sup> A. A. Thiele, *Phys. Rev. Lett.* **30**, 230 (1973).
- <sup>42</sup> J. C. Slonczewski, *J. Magn. Magn. Mater.* **12**, 108 (1979).
- <sup>43</sup> B. Hu and X. R. Wang, *Phys. Rev. Lett.* **111**, 027205 (2013).
- <sup>44</sup> H. Y. Yuan and X. R. Wang, *AIP Advances* **5**, 117104 (2015).
- <sup>45</sup> D. L. Huber, *Phys. Rev. B* **26**, 3758 (1982).
- <sup>46</sup> J. He, Z. Li, and S. Zhang, *Phys. Rev. B* **73**, 184408 (2006).
- <sup>47</sup> H. Y. Yuan and X. R. Wang, *Europhys. J. B* **88**, 214 (2015).
- <sup>48</sup> H. Y. Yuan and X. R. Wang, *Phys. Rev. B* **92**, 054419 (2015).
- <sup>49</sup> G. S. D. Beach, C. Nistor, C. Knutson, M. Tsoi, and J. L. Erskine, *Nat. Mater.* **4**, 741 (2005).
- <sup>50</sup> The width of nanowires are taken from the corresponding experiment and  $r_c = 5$  nm is fixed in this work for permalloy.
- <sup>51</sup> Y. Li and W. E. Bailey, *Phys. Rev. Lett.* **116**, 117602 (2016).
- <sup>52</sup> H. T. Nembach, J. M. Shaw, C. T. Boone, and T. J. Silva, *Phys. Rev. Lett.* **110**, 117201 (2013).
- <sup>53</sup> M. A. W. Schoen, J. M. Shaw, H. T. Nembach, M. Weiler, and T. J. Silva, *Phys. Rev. B* **92**, 184417 (2015).
- <sup>54</sup> M. Hayashi, L. Thomas, C. Rettner, R. Moriya, and S. S. P. Parkin, *Appl. Phys. Lett.* **92**, 162503 (2008).
- <sup>55</sup> S. Lepadatu, A. Vanhaverbeke, D. Atkinson, R. Allenspach, and C. H. Marrows, *Phys. Rev. Lett.* **102**, 127203 (2009).
- <sup>56</sup> M. Eltschka, M. Wötzl, J. Rhensius, S. Krzyk, U. Nowak, M. Kläui, T. Kasama, R. E. Dunin-Borkowski, L. J. Heyderman, H. J. van Driel, and R. A. Duine, *Phys. Rev. Lett.* **105**, 056601 (2010).
- <sup>57</sup> J.-Y. Chauleau, H. G. Bauer, H. S. Körner, J. Stigloher, M. Härtinger, G. Woltersdorf, and C. H. Back, *Phys. Rev. B* **89**, 020403(R) (2014).
- <sup>58</sup> J. -V. Kim, *Phys. Rev. B* **92**, 014418 (2015).